Intégrales définies

Calculer les intégrales définies suivantes

$$\int_{-1}^{1} 2x^3 - x + 1 \, dx$$

$$\int_{0}^{1} (1-x)^2 \, dx$$

$$\int_{0}^{1} e^{2u} \, du$$

$$\int_{4}^{2} t\sqrt{t^2+4} \, dt$$

$$\int_{1}^{e} \frac{\ln^2[x]}{x} \, dx$$

$$\int_{\ln(2)}^{\ln(3)} \frac{e^x}{-1+e^x} \, dx$$

$$\int_{\frac{1}{e}}^{e} (2x-1)\ln(x) \, dx$$

$$\int_{-1}^{2} 2\sqrt{4-x} \, x \, dx$$

$$\int_{-2}^{-1} \frac{t-2}{t^2-4t+3} \, dt$$

$$\int_{\frac{\pi}{4}}^{0} \mathrm{tg}(x)\left(\mathrm{tg}^2[x]+1\right) dx$$

■ Solutions

$$\int_{-1}^{1} 2x^3 - x + 1\, dx = \left[\frac{x^4}{2} - \frac{x^2}{2} + x \right]_{-1}^{1} = 2$$

$$\int_{0}^{1} (1-x)^2\, dx = \left[\frac{x^3}{3} - x^2 + x \right]_{0}^{1} = \frac{1}{3}$$

$$\int_{0}^{1} e^{2u}\, du = \left[\frac{e^{2u}}{2} \right]_{0}^{1} = \frac{1}{2}\left(-1 + e^2\right)$$

$$\int_{4}^{2} t\sqrt{t^2+4}\, dt = \left[\frac{1}{3}\left(t^2+4\right)^{3/2} \right]_{4}^{2} = \frac{8}{3}\left(2\sqrt{2} - 5\sqrt{5}\right)$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2(x)\sin(x)\, dx = \left[-\frac{1}{3}\cos^3(x) \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{1}{24}\left(-1 + 2\sqrt{2}\right)$$

$$\int_{1}^{e} \frac{\ln^2[x]}{x}\, dx = \left[\frac{\ln^3[x]}{3} \right]_{1}^{e} = \frac{1}{3}$$

$$\int_{\ln(2)}^{\ln(3)} \frac{e^x}{-1+e^x}\, dx = \left[\ln(|-1+e^x|) \right]_{\ln(2)}^{\ln(3)} = \ln(2)$$

$$\int_{\frac{1}{e}}^{e} (2x-1)\ln(x)\, dx = \left[-\frac{x^2}{2} + x + \left(x^2 - x\right)\ln(x) \right]_{\frac{1}{e}}^{e} = \frac{3 - 4e + e^4}{2e^2}$$

$$\int_{-1}^{2} 2\sqrt{4-x}\; x\, dx = \left[-\frac{4}{15}(4-x)^{3/2}(3x+8) \right]_{-1}^{2} = -\frac{4}{15}\left(28\sqrt{2} - 25\sqrt{5}\right)$$

$$\int_{-2}^{-1} \frac{t-2}{t^2-4t+3}\, dt = \left[\frac{1}{2}\ln\left(\left|t^2-4t+3\right|\right) \right]_{-2}^{-1} = -\frac{1}{2}\ln\left(\frac{6}{5}\right) - \ln\left(\frac{5}{4}\right)$$

$$\int_{\frac{\pi}{4}}^{0} \mathrm{tg}(x)\left(\mathrm{tg}^2[x] + 1\right)\, dx = \left[\frac{1}{2}\,\frac{1}{\cos^2(x)} \right]_{\frac{\pi}{4}}^{0} = -\frac{1}{2}$$