

## Exercices sur les limites : Exemples

### Indétermination $\left[\frac{0}{0}\right]$

#### ■ Exemple 1

$$\lim_{x \rightarrow \frac{1}{2}} \frac{2x^2 + 7x - 4}{2x^2 - 7x + 3} = \left[\frac{0}{0}\right]$$

$$= \lim_{x \rightarrow \frac{1}{2}} \frac{(x+4)(2x-1)}{(x-3)(2x-1)}$$

$$= \lim_{x \rightarrow \frac{1}{2}} \frac{x+4}{x-3}$$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{x+4}{x-3} = -\frac{9}{5}$$

#### ■ Exemple 2

$$\lim_{x \rightarrow 5} \frac{\sqrt{2x-1} - 3}{x^2 - 3x - 10} = \left[\frac{0}{0}\right]$$

$$= \lim_{x \rightarrow 5} \frac{(\sqrt{2x-1} - 3)(\sqrt{2x-1} + 3)}{(x-5)(x+2)(\sqrt{2x-1} + 3)}$$

$$= \lim_{x \rightarrow 5} \frac{2x - 10}{(x-5)(x+2)(\sqrt{2x-1} + 3)}$$

$$\lim_{x \rightarrow 5} \frac{2x - 10}{(x-5)(x+2)(\sqrt{2x-1} + 3)}$$

$$\lim_{x \rightarrow 5} \frac{2}{(x+2)(\sqrt{2x-1} + 3)} = \frac{1}{21}$$

### Signe de l'infini - Indétermination $\left[\frac{k}{0}\right]$

#### ■ Exemple 1

$$\lim_{x \rightarrow -1} \frac{2x-3}{x+1} = \left[\frac{-5}{0}\right]$$

$x$		-1		$\frac{3}{2}$	
$2x-3$	-	-	-	0	+
$x+1$	-	0	+	+	+
$\frac{2x-3}{x+1}$	+		-	0	+

$$\left\{ \begin{array}{l} \lim_{x \rightarrow -1} \frac{2x-3}{x+1} = +\infty \\ < \\ \lim_{x \rightarrow -1} \frac{2x-3}{x+1} = -\infty \\ > \end{array} \right.$$

## Exemple 2

$$\lim_{x \rightarrow 3} \frac{x^2 + x + 3}{x^2 - 5x + 6} = \left[ \frac{15}{0} \right]$$

$x$		2		3	
$x^2 + x + 3$	+	+	+	+	+
$x^2 - 5x + 6$	+	0	-	0	+
$\frac{x^2+x+3}{x^2-5x+6}$	+		-		+

$$\begin{cases} \lim_{x \rightarrow 3} \frac{x^2+x+3}{x^2-5x+6} = -\infty \\ < \\ \lim_{x \rightarrow 3} \frac{x^2+x+3}{x^2-5x+6} = +\infty \\ > \end{cases}$$

## ■ Exemple 3

$$\lim_{x \rightarrow -2} \frac{\sqrt{x^2 - 2x}}{x + 2} = \left[ \frac{2\sqrt{2}}{0} \right]$$

$x$		-2		0		2	
$\sqrt{x^2 - 2x}$	+	+	+	0		0	+
$x + 2$	-	0	+	+	+	+	+
$\frac{\sqrt{x^2-2x}}{x+2}$	-		+	0		0	+

$$\begin{cases} \lim_{x \rightarrow -2} \frac{\sqrt{x^2-2x}}{x+2} = -\infty \\ < \\ \lim_{x \rightarrow -2} \frac{\sqrt{x^2-2x}}{x+2} = +\infty \\ > \end{cases}$$

## ■ Exemple 4

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{3-x}$$

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{3-x} = \left[ \frac{2}{0} \right]$$

$x$		-1		3	
$\sqrt{x+1}$		0	+	+	+
$3-x$	+	+	+	0	-
$\frac{\sqrt{x+1}}{3-x}$		0	+		-

$$\begin{cases} \lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{3-x} = +\infty \\ < \\ \lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{3-x} = -\infty \\ > \end{cases}$$

Indétermination  $\left[\frac{\infty}{\infty}\right]$ 

RAPPEL:  $\sqrt{x^2} = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

## ■ Exemple 1

$$\lim_{x \rightarrow +\infty} \frac{2x-3}{x+1} = \lim_{x \rightarrow +\infty} \frac{2x}{x} = \lim_{x \rightarrow +\infty} 2 = 2$$

## ■ Exemple 2

$$\lim_{x \rightarrow -\infty} \frac{3-4x}{x^2+3x+1} = \lim_{x \rightarrow -\infty} \frac{-4x}{x^2} = \lim_{x \rightarrow -\infty} -\frac{4}{x} = 0$$

## ■ Exemple 3

$$\lim_{x \rightarrow -\infty} \frac{x^2-5x}{3x-2} = \lim_{x \rightarrow -\infty} \frac{x^2}{3x} = \lim_{x \rightarrow -\infty} \frac{x}{3} = -\infty$$

## ■ Exemple 4

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+3x+1}}{2x-3} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2}}{2x} = \lim_{x \rightarrow -\infty} \frac{-x}{2x} = -\frac{1}{2}$$

## ■ Exemple 5

$$\lim_{x \rightarrow -\infty} \sqrt{x^2+3x+1} - x = \lim_{x \rightarrow -\infty} \sqrt{x^2} - x = \lim_{x \rightarrow -\infty} -2x = +\infty$$

## ■ Exemple 6

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$$\lim_{x \rightarrow -\infty} \sqrt{5 - 2x + x^2} + x = [\infty + (-\infty)]$$

$$= \lim_{x \rightarrow -\infty} \frac{(\sqrt{5 - 2x + x^2} - x)(\sqrt{5 - 2x + x^2} + x)}{\sqrt{5 - 2x + x^2} - x}$$

$$= \lim_{x \rightarrow -\infty} \frac{5 - 2x}{\sqrt{5 - 2x + x^2} - x}$$

$$= \lim_{x \rightarrow -\infty} \frac{5 - 2x}{\sqrt{5 - 2x + x^2} - x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2x}{\sqrt{x^2} - x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2x}{|x| - x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2x}{-2x}$$

$$\lim_{x \rightarrow -\infty} x + \sqrt{x^2 - 2x + 5} = 1$$