Exercices - Calcul de limites

- Calculer les limites suivantes:

1) $\lim\limits_{x\to 1} \dfrac{2x^2-x-10}{-x^2+4x-3}$

2) $\lim\limits_{x\to -3} \dfrac{-3x^3-23x^2-37x+15}{5x^2+14x-3}$

3) $\lim\limits_{x\to \sqrt{2}} \dfrac{x^4-5x^2+6}{\sqrt{2}\,x^2-3x+\sqrt{2}}$

4) $\lim\limits_{x\to -4} \dfrac{-6x^2+x+1}{x^2+2x-8}$

5) $\lim\limits_{x\to +\infty} \dfrac{-6x^2+x+1}{x^2+2x-8}$

6) $\lim\limits_{x\to -\infty} \sqrt{\dfrac{3-5x}{x^2-5x+3}}$

7) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{3x^2-5x+8}}{\sqrt{5x^2-4}}$

8) $\lim\limits_{x\to -\infty} \dfrac{x-4}{\sqrt{2x^2+1}}$

9) $\lim\limits_{x\to 7} \dfrac{2\sqrt{x-3}-\sqrt{2x+2}}{5-\sqrt{3x+4}}$

10) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{3x-2}-\sqrt[3]{x}}{1-x^2}$

- Solutions

1)

$$\lim_{x\to 1} \frac{2x^2-x-10}{-x^2+4x-3} = \left[\frac{-9}{0}\right]$$

x		-2		1		$\frac{5}{2}$		3	
$2x^2-x-10$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$-x^2+4x-3$	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$
$\frac{2x^2-x-10}{-x^2+4x-3}$	$-$	0	$+$	$\|$	$-$	0	$+$	$\|$	$-$

$$\begin{cases} \lim\limits_{\substack{x\to 1\\ <}} \frac{2x^2-x-10}{-x^2+4x-3} = +\infty \\ \lim\limits_{\substack{x\to 1\\ >}} \frac{2x^2-x-10}{-x^2+4x-3} = -\infty \end{cases}$$

2)

$$\lim_{x\to -3} \frac{-3x^3-23x^2-37x+15}{5x^2+14x-3} = \left[\frac{0}{0}\right]$$

	-3	-23	-37	15
-3		9	42	-15
	-3	-14	5	0

$$= \lim_{x\to -3} \frac{-(x+3)(x+5)(3x-1)}{(x+3)(5x-1)}$$

$$= \lim_{x\to -3} -\frac{(x+5)(3x-1)}{5x-1}$$

$$= -\frac{5}{4}$$

3) *math 6 b*

$$\lim_{x\to \sqrt{2}} \frac{x^4-5x^2+6}{\sqrt{2}\,x^2-3x+\sqrt{2}} = [\frac{0}{0}]$$

$$= \lim_{x\to \sqrt{2}} \frac{(x^2-3)(x^2-2)}{\sqrt{2}\,x^2-3x+\sqrt{2}}$$

$$= \lim_{x\to \sqrt{2}} \frac{(x^2-3)(x^2-2)}{\sqrt{2}\,x^2-3x+\sqrt{2}} = \lim_{x\to \sqrt{2}} \frac{(x^2-3)\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{\left(x-\sqrt{2}\right)\left(\sqrt{2}\,x-1\right)}$$

$$= \lim_{x\to \sqrt{2}} \frac{(x^2-3)\left(x+\sqrt{2}\right)}{\sqrt{2}\,x-1} = -2\sqrt{2}$$

4)

$$\lim_{x\to -4} \frac{-6x^2+x+1}{x^2+2x-8} = [\frac{-99}{0}]$$

x		-4		$-\frac{1}{3}$		$\frac{1}{2}$		2	
$-6x^2+x+1$	$-$	$-$	$-$	0	$+$	0	$-$	$-$	$-$
x^2+2x-8	$+$	0	$-$	$-$	$-$	$-$	$-$	0	$+$
$\frac{-6x^2+x+1}{x^2+2x-8}$	$-$	$\|$	$+$	0	$-$	0	$+$	$\|$	$-$

$$\begin{cases} \lim\limits_{\substack{x\to -4 \\ <}} \frac{-6x^2+x+1}{x^2+2x-8} = -\infty \\ \lim\limits_{\substack{x\to -4 \\ >}} \frac{-6x^2+x+1}{x^2+2x-8} = +\infty \end{cases}$$

5)

$$\lim_{x\to +\infty} \frac{-6x^2+x+1}{x^2+2x-8} = \lim_{x\to +\infty} \frac{-6x^2}{x^2} = -6$$

6)

$$\lim_{x\to -\infty} \sqrt{\frac{3-5x}{x^2-5x+3}} = \lim_{x\to -\infty} \sqrt{\frac{-5x}{x^2}} = \lim_{x\to -\infty} \sqrt{\frac{-5}{x}} = 0$$

7)

$$\lim_{x\to -\infty} \frac{\sqrt{3x^2-5x+8}}{\sqrt{5x^2-4}} = \lim_{x\to -\infty} \frac{\sqrt{3}\,\sqrt{x^2}}{\sqrt{5}\,\sqrt{x^2}} = \lim_{x\to -\infty} \frac{-\sqrt{3}\,x}{-\sqrt{5}\,x} = \sqrt{\frac{3}{5}}$$

8)

$$\lim_{x\to -\infty} \frac{x-4}{\sqrt{2x^2+1}} = \lim_{x\to -\infty} \frac{x}{\sqrt{2}\,\sqrt{x^2}} = \lim_{x\to -\infty} \frac{x}{-\sqrt{2}\,x} = -\frac{1}{\sqrt{2}}$$

9)

$$\lim_{x\to 7} \frac{2\sqrt{x-3}-\sqrt{2x+2}}{5-\sqrt{3x+4}} = [\frac{0}{0}]$$

$$= \lim_{x\to 7} \frac{\left(2\sqrt{x-3}-\sqrt{2x+2}\right)\left(2\sqrt{x-3}+\sqrt{2x+2}\right)\left(\sqrt{3x+4}+5\right)}{\left(2\sqrt{x-3}+\sqrt{2x+2}\right)\left(5-\sqrt{3x+4}\right)\left(\sqrt{3x+4}+5\right)}$$

$$= \lim_{x\to 7} \frac{2(x-7)\left(\sqrt{3x+4}+5\right)}{\left(2\sqrt{x-3}+\sqrt{2x+2}\right)(25-3x-4)} = \lim_{x\to 7} \frac{2(x-7)\left(\sqrt{3x+4}+5\right)}{\left(2\sqrt{x-3}+\sqrt{2x+2}\right)3(7-x)}$$

$$= \lim_{x\to 7} \frac{-2\left(\sqrt{3x+4}+5\right)}{3\left(2\sqrt{x-3}+\sqrt{2x+2}\right)} = \frac{-20}{24} = \frac{-5}{6}$$

10) *math 6 b*

$$\lim_{x\to 1} \frac{\sqrt[3]{3x-2}-\sqrt[3]{x}}{1-x^2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \lim_{x\to 1} \frac{\sqrt[3]{3x-2}-\sqrt[3]{x}}{1-x^2} \frac{\sqrt[3]{(3x-2)^2}+\sqrt[3]{3x-2}\ \sqrt[3]{x}+\sqrt[3]{x^2}}{\sqrt[3]{(3x-2)^2}+\sqrt[3]{3x-2}\ \sqrt[3]{x}+\sqrt[3]{x^2}}$$

$$= \lim_{x\to 1} \frac{3x-(2+x)}{(1-x^2)\left(\sqrt[3]{(3x-2)^2}+\sqrt[3]{3x-2}\ \sqrt[3]{x}+\sqrt[3]{x^2}\right)}$$

$$= \lim_{x\to 1} \frac{2(x-1)}{(1-x)(1+x)\left(\sqrt[3]{(3x-2)^2}+\sqrt[3]{3x-2}\ \sqrt[3]{x}+\sqrt[3]{x^2}\right)}$$

$$= \lim_{x\to 1} \frac{-2}{(1+x)\left(\sqrt[3]{(3x-2)^2}+\sqrt[3]{3x-2}\ \sqrt[3]{x}+\sqrt[3]{x^2}\right)} = \frac{-2}{6} = \frac{-1}{3}$$