
DÉTERMINER LE DOMAINE DE DÉFINITION DES FONCTIONS SUIVANTES:

- 1) $f(x) = \text{Arcsin}(3 - x^2)$
- 2) $f(x) = \text{Arctg}\left(\frac{1}{x-2}\right)$
- 3) $f(x) = \text{Arccos}(x^2 - 2x)$
- 4) $f(x) = \frac{2}{\text{Arcsin}(3x)}$
- 5) $f(x) = \frac{\text{Arccos}(2x)}{\text{Arctg}\left(\frac{x}{2}\right)}$
- 6) $f(x) = \text{Arcsin}(-x^2 - 2x + 1)$
- 7) $f(x) = \text{Arcsin}(2 - 5x)$
- 8) $f(x) = \text{Arccos}\left(\frac{1}{x}\right)$
- 9) $f(x) = \text{Arcsin}(3x) - \text{Arccos}(2x)$
- 10) $f(x) = \text{Arctg}(6x^2 - 3x)$
- 11) $f(x) = \text{Arccos}\left(\frac{x}{x+1}\right)$
- 12) $f(x) = \text{Arctg} \frac{2x+1}{x+1}$

SOLUTIONS DÉTAILLÉES

■ 1) $f(x) = \text{Arcsin}(3 - x^2)$

$$-1 \leq 3 - x^2 \leq 1$$

$$-1 \leq 3 - x^2 \quad \text{et} \quad 3 - x^2 \leq 1$$

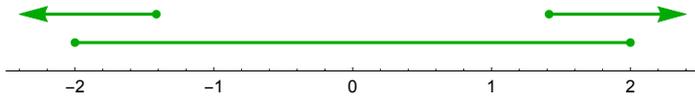
$$x^2 - 4 \leq 0 \quad \text{et} \quad 2 - x^2 \leq 0$$

$$x^2 - 4 \leq 0 \quad \text{et} \quad 2 - x^2 \leq 0$$

x		-2		2	
$x^2 - 4$	+	0	-	0	+

x		$-\sqrt{2}$		$\sqrt{2}$	
$2 - x^2$	-	0	+	0	-

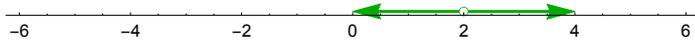
$$([-2; 2]) \cap (\leftarrow; -\sqrt{2}] \cup [\sqrt{2}; \rightarrow) = [-2; -\sqrt{2}] \cup [\sqrt{2}; 2]$$



$$\text{dom } f = [-2; -\sqrt{2}] \cup [\sqrt{2}; 2]$$

■ 2) $f(x) = \text{Arctg}\left(\frac{1}{x-2}\right)$

$$x - 2 \neq 0 \Leftrightarrow x \neq 2 \Leftrightarrow x \in \mathbb{R} \setminus \{2\}$$



$$\text{dom } f = \mathbb{R} \setminus \{2\}$$

■ 3) $f(x) = \text{Arccos}(x^2 - 2x)$

$$-1 \leq x^2 - 2x \leq 1$$

$$-1 \leq x^2 - 2x \quad \text{et} \quad x^2 - 2x \leq 1$$

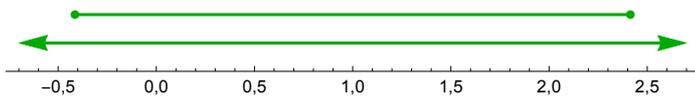
$$-x^2 + 2x - 1 \leq 0 \quad \text{et} \quad x^2 - 2x - 1 \leq 0$$

$$-x^2 + 2x - 1 \leq 0 \quad \text{et} \quad x^2 - 2x - 1 \leq 0$$

x		1	
$-x^2 + 2x - 1$	-	0	-

x		$1 - \sqrt{2}$		$1 + \sqrt{2}$	
$x^2 - 2x - 1$	+	0	-	0	+

$$(\mathbb{R}) \cap ([1 - \sqrt{2}; 1 + \sqrt{2}]) = [1 - \sqrt{2}; 1 + \sqrt{2}]$$



$$\text{dom } f = [1 - \sqrt{2}; 1 + \sqrt{2}]$$

■ 4) $f(x) = \frac{2}{\text{Arcsin}(3x)}$

$$\text{Arcsin}(3x) \neq 0 \Leftrightarrow x \neq 0$$

$$\text{et } -1 \leq 3x \leq 1$$

$$-1 \leq 3x \quad \text{et} \quad 3x \leq 1$$

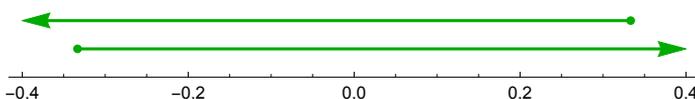
$$-3x - 1 \leq 0 \quad \text{et} \quad 3x - 1 \leq 0$$

$$-3x - 1 \leq 0 \quad \text{et} \quad 3x - 1 \leq 0$$

x		$-\frac{1}{3}$	
$-3x - 1$	+	0	-

x		$\frac{1}{3}$	
$3x - 1$	-	0	+

$$\left(-\frac{1}{3}; \rightarrow\right) \cap \left(\leftarrow; \frac{1}{3}\right) = \left[-\frac{1}{3}; \frac{1}{3}\right]$$



$$\text{dom } f = \left[-\frac{1}{3}; 0[\cup]0; \frac{1}{3}\right]$$

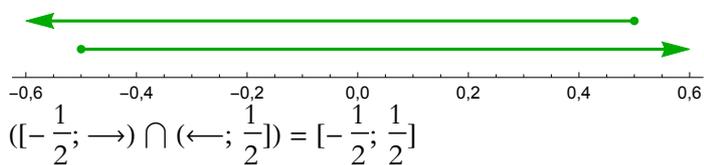
■ 5) $f(x) = \frac{\text{Arccos}(2x)}{\text{Arctg}\left(\frac{x}{2}\right)}$

$\text{Arctg}\left(\frac{x}{2}\right) \neq 0 \Leftrightarrow x \neq 0 \Leftrightarrow x \in \mathbb{R} \setminus \{0\}$

$-1 \leq 2x \leq 1$

$-1 \leq 2x$ et $2x \leq 1$

$x \geq \frac{-1}{2}$ et $x \leq \frac{1}{2}$



$\text{dom } f = \left[-\frac{1}{2}; 0[\cup]0; \frac{1}{2}\right]$

■ 6) $f(x) = \text{Arcsin}(-x^2 - 2x + 1)$

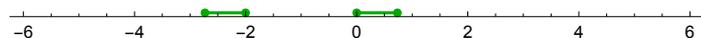
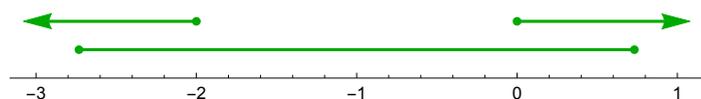
$-1 \leq -x^2 - 2x + 1 \leq 1$

$-1 \leq -x^2 - 2x + 1$ et $-x^2 - 2x + 1 \leq 1$

$x^2 + 2x - 2 \leq 0$ et $-x^2 - 2x \leq 0$

x		$-1 - \sqrt{3}$		$-1 + \sqrt{3}$	
$x^2 + 2x - 2$	+	0	-	0	+

x		-2		0	
$-x^2 - 2x$	-	0	+	0	-



$(\left[-1 - \sqrt{3}; -1 + \sqrt{3}\right]) \cap (\leftarrow; -2] \cup [0; \rightarrow) = \left[-1 - \sqrt{3}; -2\right] \cup [0; -1 + \sqrt{3}]$

$\text{dom } f = \left[-1 - \sqrt{3}; -2\right] \cup [0; -1 + \sqrt{3}]$

■ 7) $f(x) = \text{Arcsin}(2 - 5x)$

$-1 \leq 2 - 5x \leq 1$

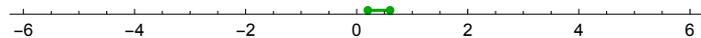
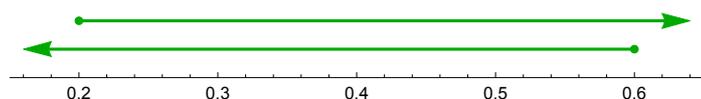
$-1 \leq 2 - 5x$ et $2 - 5x \leq 1$

$5x - 3 \leq 0$ et $1 - 5x \leq 0$

x		$\frac{3}{5}$	
$5x - 3$	-	0	+

x		$\frac{1}{5}$	
$1 - 5x$	+	0	-

$(\leftarrow; \frac{3}{5}) \cap (\frac{1}{5}; \rightarrow) = \left[\frac{1}{5}; \frac{3}{5}\right]$



$\text{dom } f = \left[\frac{1}{5}; \frac{3}{5}\right]$

■ 8) $f(x) = \text{Arccos}\left(\frac{1}{x}\right)$

$x \neq 0$ et

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 $-1 \leq \frac{x}{1-x} \leq 1$

$-1 \leq \frac{1}{x}$ et $\frac{1}{x} \leq 1$

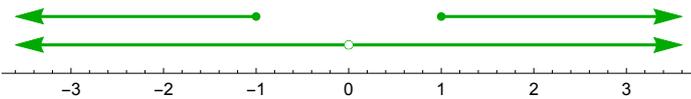
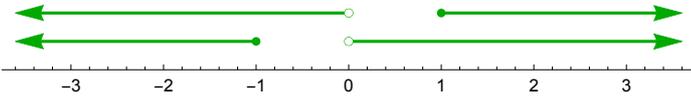
$-1 - \frac{1}{x} \leq 0$ et $\frac{1}{x} - 1 \leq 0$

$\frac{-x-1}{x} \leq 0$ et $\frac{1-x}{x} \leq 0$

x		-1		0	
$\frac{-x-1}{x}$	-	0	+		-

x		0		1	
$\frac{1-x}{x}$	-		+	0	-

$(\leftarrow; -1] \cup]0; \rightarrow) \cap (\leftarrow; 0[\cup]1; \rightarrow) = \leftarrow; -1] \cup]1; \rightarrow$



$\text{dom } f = \leftarrow; -1] \cup]1; \rightarrow$

■ 9) $f(x) = \text{Arcsin}(3x) - \text{Arccos}(2x)$

$-1 \leq 2x \leq 1$

$-1 \leq 2x$ et $2x \leq 1$

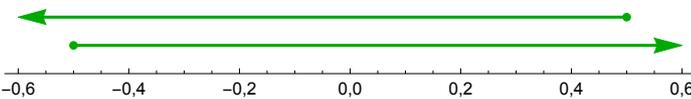
$-2x-1 \leq 0$ et $2x-1 \leq 0$

$-2x-1 \leq 0$ et $2x-1 \leq 0$

x		$-\frac{1}{2}$	
$-2x-1$	+	0	-

x		$\frac{1}{2}$	
$2x-1$	-	0	+

$([-\frac{1}{2}; \rightarrow) \cap (\leftarrow; \frac{1}{2}]) = [-\frac{1}{2}; \frac{1}{2}]$



$-1 \leq 3x \leq 1$

$-1 \leq 3x$ et $3x \leq 1$

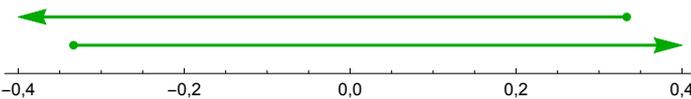
$-3x-1 \leq 0$ et $3x-1 \leq 0$

$-3x-1 \leq 0$ et $3x-1 \leq 0$

x		$-\frac{1}{3}$	
$-3x-1$	+	0	-

x		$\frac{1}{3}$	
$3x-1$	-	0	+

$([-\frac{1}{3}; \rightarrow) \cap (\leftarrow; \frac{1}{3}]) = [-\frac{1}{3}; \frac{1}{3}]$



$\text{dom } f = [-\frac{1}{3}; \frac{1}{3}]$

■ 10) $f(x) = \text{Arctg}(6x^2 - 3x)$

$\text{dom } f = \mathbb{R}$

■ 11) $f(x) = \text{Arccos}\left(\frac{x}{x+1}\right)$

$x+1 \neq 0 \Leftrightarrow x \neq -1 \Leftrightarrow x \in \mathbb{R} \setminus \{-1\}$

$-1 \leq \frac{x}{x+1} \leq 1$

$-1 \leq \frac{x}{x+1}$ et $\frac{x}{x+1} \leq 1$

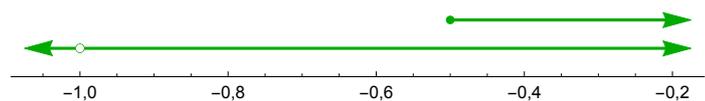
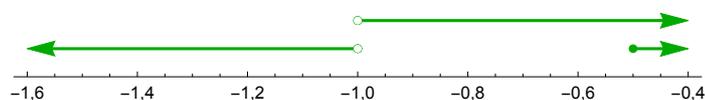
$-\frac{x}{x+1} - 1 \leq 0$ et $\frac{x}{x+1} - 1 \leq 0$

$\frac{-2x-1}{x+1} \leq 0$ et $-\frac{1}{x+1} \leq 0$

x		-1		$-\frac{1}{2}$	
$\frac{-2x-1}{x+1}$	-		+	0	-

x		-1	
$-\frac{1}{x+1}$	+		-

$(\leftarrow; -1[\cup [-\frac{1}{2}; \rightarrow) \cap (]-1; \rightarrow) = [-\frac{1}{2}; \rightarrow)$



$\text{dom } f = [-\frac{1}{2}; \rightarrow)$

■ 12) $f(x) = \text{Arctg} \frac{2x+1}{x+1}$

$x+1 \neq 0 \Leftrightarrow x \neq -1 \Leftrightarrow x \in \mathbb{R} \setminus \{-1\}$



$\text{dom } f = \mathbb{R} \setminus \{-1\}$