Fonctions cyclométriques - dérivation

■ Formules de dérivation

$$(\text{Arcsin}(x))' = \frac{1}{\sqrt{1-x^2}} \qquad\qquad (\text{Arcsin}(u))' = \frac{u'}{\sqrt{1-u^2}}$$

$$(\text{Arccos}(x))' = \frac{-1}{\sqrt{1-x^2}} \qquad\qquad (\text{Arccos}(u))' = \frac{-u'}{\sqrt{1-u^2}}$$

$$(\text{Arctg}(x))' = \frac{1}{1+x^2} \qquad\qquad (\text{Arctg}(u))' = \frac{u'}{1+u^2}$$

■ Exercices: dériver les fonctions suivantes:

a) $f(x) = \text{Arcsin}(2\,x)$

b) $f(x) = \text{Arccos}(x-1)$

c) $f(x) = \text{Arccos}(x^2-1)$

d) $f(x) = \dfrac{1}{\text{Arcsin}(x)}$

e) $f(x) = \text{Arccos}^2(x)$

f) $f(x) = \text{Arctg}(\sqrt{x}\,)$

g) $f(x) = \sqrt{\text{Arctg}(x)}$

h) $f(x) = x\,\text{Arcsin}(x) + \sqrt{1-x^2}$

i) $f(x) = \text{Arccos}\left(\dfrac{1}{x}\right)$

j) $f(x) = x\,\text{Arctg}(x) - \dfrac{1}{2}\ln(x^2+1)$

a) $f'(x) = \dfrac{2}{\sqrt{1 - 4\,x^2}}$

b) $f'(x) = \dfrac{-1}{\sqrt{2\,x - x^2}}$

c) $f'(x) = \dfrac{-2\,x}{\sqrt{2\,x^2 - x^4}}$

d) $f'(x) = -\dfrac{1}{\sqrt{1 - x^2}\,\operatorname{Arcsin}^2(x)}$

e) $f'(x) = -\dfrac{2\operatorname{Arccos}(x)}{\sqrt{1 - x^2}}$

f) $f'(x) = \dfrac{1}{2\sqrt{x}\,(x + 1)}$

g) $f'(x) = \dfrac{1}{\left(2\,x^2 + 2\right)\sqrt{\operatorname{Arctg}(x)}}$

h) $f'(x) = \operatorname{Arcsin}(x)$

i) $f'(x) = \dfrac{-1}{\sqrt{1 - \left(\frac{1}{x}\right)^2}}\left(\dfrac{1}{x}\right)' = \dfrac{-1}{\sqrt{1 - \frac{1}{x^2}}}\dfrac{-1}{x^2} = \dfrac{1}{x^2\sqrt{\frac{x^2-1}{x^2}}} = \dfrac{1}{x\sqrt{x^2 - 1}}$

j) $f'(x) = \operatorname{Arctg}(x) + \dfrac{x}{1 + x^2} - \dfrac{1}{2}\dfrac{2\,x}{1 + x^2} = \operatorname{Arctg}(x)$